

Foundations of Discrete Mathematics
COT 2104

Practice 3

1. Use set builder notation to give a description of each of these sets.
 - a. $\{0, 3, 6, 9, 12\}$
 - b. $\{-3, -2, -1, 0, 1, 2, 3\}$
 - c. $\{m, n, o, p\}$
2. Suppose that $A = \{2, 4, 6\}$, $B = \{2, 6\}$, $C = \{4, 6\}$, and $D = \{4, 6, 8\}$. Determine which of these sets are subsets of which other of these sets are.
3. Determine whether each of these statements is true or false.
 - a. $\{\emptyset\} \in \{\emptyset\}$
 - b. $\{\emptyset\} \in \{\{\emptyset\}\}$
 - c. $\{\{\emptyset\}\} \subseteq \{\emptyset, \{\emptyset\}\}$
4. Use a Venn diagram to illustrate the relationship $A \subseteq B$ and $B \subseteq C$.
5. Find two sets A and B such that $A \in B$ and $A \subseteq B$.
6. Find the cardinality of each of these sets.
 - a) \emptyset
 - b) $\{\emptyset\}$
 - c) $\{\emptyset, \{\emptyset\}\}$
 - d) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$
7. Determine whether each of these sets is the power set of a set.
 - a) \emptyset
 - b) $\{\emptyset, \{a\}\}$
 - c) $\{\emptyset, \{a\}, \{\emptyset, a\}\}$
 - d) $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
8. Let $A = \{a, b, c\}$, $B = \{x, y\}$. Find $A \times B$.
9. Show that $A \times B \neq B \times A$, when A and B are nonempty, unless $A = B$.

10. Translate each of these quantifications into English and determine its truth value.

- a) $\exists x \in \mathbf{R} (x^3 = -1)$.
- b) $\exists x \in \mathbf{Z} (x + 1 > x)$.
- c) $\exists x \in \mathbf{Z} (x - 1 \in \mathbf{Z})$.
- d) $\exists x \in \mathbf{Z} (x^2 \in \mathbf{Z})$.

11. Suppose that A is the set of sophomores at your school and B is the set of students in discrete mathematics at your school. Express each of these sets in terms of A and B.

- a) The set of sophomores taking discrete mathematics in your school.
- b) The set of sophomores at your school who are not taking discrete mathematics.
- c) The set of students at your school who either are sophomores or are taking discrete mathematics.
- d) The set of students at your school who either are not sophomores or are not taking discrete mathematics.

12. Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find

- a) $A \cup B$
- b) $A \cap B$
- c) $A \setminus B$
- d) $B \setminus A$

13. Let A be a set. Show that

- a) $A \cup \emptyset = A$
- b) $A \cap \emptyset = \emptyset$
- c) $A \cup A = A$
- d) $A \setminus \emptyset = A$

14. Let $A = \{0, 2, 4, 6, 8, 10\}$, $B = \{0, 1, 2, 3, 4, 5, 6\}$, and $C = \{4, 5, 6, 7, 8, 9, 10\}$. Find

- a) $A \cap B \cap C$
- b) $(A \cup B) \cap C$

15. Let A and B be sets. Show that

- a) $(A \cap B) \subseteq A$
- b) $A \setminus B \subseteq A$

16. Let A , B and C be sets. Show that

a) $(A \cup B) \subseteq (A \cup B \cup C)$

b) $(A \setminus B) \setminus C \subseteq A \setminus C$

17. Find the symmetric difference of $\{1, 3, 5\}$ and $\{1, 2, 3\}$

18. Draw a Venn diagram for the symmetric difference of the sets A and B .

19. Show that $A \oplus B = (A \setminus B) \cup (B \setminus A)$

20. Determine, with reasons whether or not the following defines an equivalence relation on the set A .

a) A is the set of all lines in the plane; $a \sim b$ if and only if a is perpendicular to b .